perpendicular and parallel to the plane of the crack, respectively; t, time; t_0, time when the process j_k begins; G_k, Cartesian components of the current density and electric field vectors averaged over the volume; σ_{kl}, components of the stress tensor; n_k, components of the vector of the unit normal to the crack; ϑ and ψ, spherical width and length specifying the direction of this vector; $Z = \{a, c_0, t_0, p_0, \varepsilon_c, \lambda_c\}$; p_0, initial value of p; $\Omega = \{\vartheta, \psi\}$, $f(Z, \Omega)$, distribution function; N, number of cracks per unit volume; $\bar{a}^3$, root mean cubic radius of the crack; and ω, angular frequency of the electric field oscillations.

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NUMERICAL SOLUTION OF A NONLINEAR POISSON EQUATION

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A method that is convenient for practical applications is proposed for solving the nonlinear Poisson equation.

A particular class of problems for heat exchange and for magnetohydrodynamics leads to the solution of the Poisson equation with a substantial nonlinearity on the right-hand side.

We consider the Dirichlet problem for Poisson's equation

$$\Delta u(x, y) = f(x, y, u), \quad u|_\Gamma = \varphi(x, y). \tag{1}$$

To simplify the discussion, we assume the region to be rectangular. Using quasilinearization [1] we construct the following iterational process:

$$\Delta v - f'_u(w)\, v = f(w) - f'_u(w)\, w, \tag{2}$$

where $w = u^{(n)}$; $v = u^{(n+1)}$; n is the number of the iteration.

The iterational process (2) ensures quadratic convergence for the condition of exact solution of (2) with fixed right-hand side for each iteration [1].

To determine the values of v for each iteration we use a method of incomplete factorization, similar to that described in [2]. But unlike what was assumed in [2] the splitting of the initial difference operator is represented in the form of the composition of two operators with variable coefficients.

We consider the difference analog of Eq. (2)

$$\Lambda_h v^m = q^m(v) + O(h^3), \tag{3}$$

where m is the index of the iteration for solution of the n-th of Eqs. (2); q^m is the right-hand side of (2) with correction, ensuring the required order of approximation; h equals the maximum of the steps h_x and h_y along the horizontal and vertical directions.

On a nine-point pattern we represent the solution of (3) in the form

$$\begin{gathered} \alpha_{ij} v^m_{ij} + v^m_{i-1,j} + \alpha_{ij} v^m_{i+1,j} = z^m_{ij}; \\ a_{ij} z^m_{i,j+1} + b_{ij} z^m_{ij} + a_{ij} z^m_{i,j-1} = q^m_{ij}, \end{gathered} \tag{4}$$

where the lower indices have the usual meaning. The difference operator on the left side of (3) can easily be transformed by two successive pivots with total number of operations $O(N)$, where N is the number of points of the grid.

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To determine the coefficients α, a, and b and the right-hand side of (3) with the help of the ordinary procedure of expansion of the terms of the difference operator Λ_h in a Taylor series we construct the system of equations

$$\begin{cases} (2a_{ij} + b_{ij})(2\alpha_{ij} + 1) = \Phi_{ij}; \\ a_{ij}(2\alpha_{ij} + 1) = h_x^{-2}; \\ (2a_{ij} + b_{ij})\alpha_{ij} = h_y^{-2}; \end{cases} \tag{5}$$

$$\Phi_{ij} = \max\{f'_u(w_{ij}),\ -\varkappa(h_x h_y)^{-1}\},$$

where $\varkappa$ is a negative parameter proportional to h in absolute value.

We determine the function q_{ij} from the condition that the difference scheme (3) satisfy the approximation (2) with third-order accuracy:

$$q_{ij}^m = l_{ij} + (\Phi_{ij} - d_{ij})\left(v_{ij}^{m-1} + \frac{1}{12}h_x^2\Lambda_2(v^{m-1}d)_{ij} + \right.$$

$$\left. + \frac{1}{12}h_y^2\Lambda_3(v^{m-1}d)_{ij}\right) - \left(\frac{1}{\Phi_{ij}} + \frac{1}{12}h_x^2 + \frac{1}{12}h_y^2\right)\left(\frac{1}{12}h_y^2\right)\Lambda_1 v_{ij}^{m-1} +$$

$$+ a_{ij}(\alpha_{i+1,j} - \alpha_{ij})(v_{i+1,j-1}^{m-1} + v_{i+1,j+1}^{m-1}) + a_{ij}(\alpha_{i-1,j} - \alpha_{ij})(v_{i-1,j+1}^{m-1} + v_{i-1,j-1}^{m-1}) + O\left(\frac{h_x h_y}{\Phi_{ij}}\right); \tag{6}$$

$$\tau_{ij} = f(x_i, y_j, w_{ij}) - f'_u(x_i, y_j, w_{ij})\,w_{ij};$$

$$l_{ij} = \tau_{ij} + \frac{1}{12}\Lambda_2\tau_{ij}h_x^2 + \frac{1}{12}\Lambda_3\tau_{ij}h_y^2;$$

$$d_{ij} = f'_u(x_i, y_j, w_{ij}).$$

The difference operators Λ_1, Λ_2, and Λ_3 to the second order approximate the operators $\partial^4/\partial x^2\partial y^2$, $\partial^2/\partial x^2$, and $\partial^2/\partial y^2$, respectively.

From (5), to determine the coefficients of operator (3) we obtain the expressions

$$\begin{cases} \alpha_{ij} = -T_{ij}^{-1}; \\ a_{ij} = T_{ij}\Phi_{ij}^{-1}(h_x h_y)^{-2}; \\ b_{ij} = T_{ij}(T_{ij} - 4)\Phi_{ij}^{-1}(h_x h_y)^{-2}; \end{cases} \tag{7}$$

$$T_{ij} = \Phi_{ij}h_y^2 + 2.$$

It is not difficult to verify that the conditions of good conditionality of the difference operator are satisfied. Equation (2) is solved for the n-th iteration by carrying out an iterational solution of its difference analog (3):

$$\Lambda_h v^m = q^m(\Phi,\ v^{m-1},\ f,\ w).$$

We note that for $d_{ij} \gg -\varkappa(h_x h_y)^{-1}$, i.e., for large nonlinearity, the difference operator (3) approximates (2) with high accuracy, which is verified by numerical experiments. In numerical calculations for $\varkappa = -0.25h$, the Poisson equation with fixed right-hand side was integrated 3-4 times for $n \leq 3$; subsequently, depending on the convergence for one external iteration we arrived at one internal iteration.

We present two examples of calculations according to the proposed method.

Example 1. $\Delta u = \exp(u)$, $u|_\Gamma = 10$, $0 \leqslant x \leqslant 0.5$, $0 \leqslant y \leqslant 0.25$. On a 32 × 16 grid with 26 internal iterations we obtained a difference between iterations not exceeding 0.00001.

For a boundary value of the function equal to 8, the process converged with the same accuracy for 16 iterations. A change in the initial approximation from 6 to 10 was not reflected in the first five decimal places of the result.

Example 2. $\Delta u = -0.5u - 0.5u^{-3}$, $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 1$. The boundary conditions were chosen so that $u = \sqrt{\sin(x + y)}$ was a solution. On an 8 × 8 grid, a solution in terms of 16 internal iterations was obtained with the same accuracy as in [3]. The initial approximation equals zero. The calculation took less than a minute on a BÉSM-4 computer.

Numerical experiments carried out with a wide class of functions showed results similar to those above and confirmed the reliability and effectiveness of the method.

NOTATION

Δ, Laplacian operator; $\Delta u = \partial^2 u/\partial x^2 + \partial^2 u/\partial y^2$; Γ, boundary of the rectangle.

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FINITE-ELEMENT CALCULATIONS ON NONSTATIONARY HEAT TRANSFER

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A finite-element technique has been used in solving a boundary-value problem for a two-dimensional nonstationary turbulent-diffusion equation.

The deposition and transport of particles in a flow of liquid can be described by a turbulent-diffusion equation if the concentration of the solid is low and the particles are sufficiently small. Rose [1] has defined the limits to the application of the diffusion theory with regard to particle size by experiment.

The models of [2,3] are relevant to the description of these processes, and some features of these are used here. The model of [3] describes the steady-state deposition of a solid material in a planar semiinfinite channel in the form of a boundary-value problem for a stationary equation in turbulent diffusion. A numerical solution was obtained by finite-difference methods and this is compared with experiment. Other studies [4-6] deal with models for water quality, in which the equations of hydrodynamics and turbulent diffusion are employed.

There are also other discussions [7-9] of nonstationary equations for turbulent diffusion; it has been suggested [8,9] that Galerkin's method should be used together with the finite-element technique, and the relevant systems of equations have been derived, but numerical treatments have been given only for the one-dimensional case [9] and for the two-dimensional case but neglecting convective terms [8]. In [7] we find a solution to a two-dimensional boundary-value problem subject to homogeneous Dirichlet conditions on the assumption that the turbulent-diffusion coefficients are constants and that there is a source of the minor component within the region only at the start.

Here we consider a model for the transport and deposition of a material suspended in a planar flow; we assume that the velocity components and the turbulent-diffusion coefficients are known functions of time and the coordinates, in which case the model can be represented as a boundary-value problem:

$$\frac{\partial c'}{\partial t'} + U(x,\ z,\ t')\,\frac{\partial \sigma'}{\partial x} + W(x,\ z,\ t')\,\frac{\partial c'}{\partial z} + \omega'\,\frac{\partial c'}{\partial z} =$$

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